Preliminary Inconclusive Hint of Evidence Against Optimal Fine Tuning of the Cosmological Constant for Maximizing the Fraction of Baryons Becoming Life [*]

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Abstract

The effective coupling 'constants' of physics, especially the cosmological constant, are observed to have highly biophilic values. If this is not a hugely improbable accident, or a consequence of some mysterious logical necessity or of some simple principle of physics, it might be explained as a consequence either of an observership selection principle within a multiverse of many sets of effective coupling constants, or else of some biophilic principle that fine tunes the constants of physics to optimize life. Here a very preliminary inconclusive hint of evidence is presented against the hypothesis of optimal fine tuning of the cosmological constant by a biophilic principle that would maximize the fraction of baryons that form living organisms or observers.

1 Introduction

It has long been recognized that many of the apparent constants of physics are observed to take values that are much more biophilic (in the sense of being conducive to life and observership) than values significantly different are believed to be [1, 2, 3, 4, 5, 6, 7]. For example, the cosmological constant (or dark energy density) that quantifies the gravitational repulsion of empty space is roughly 122 orders of magnitude smaller than the Planck value, but if it were just a few orders of magnitude larger than its tiny positive observed value [8, 9], with the other constants of physics kept the same, life as we know it would appear to be very difficult.

A partial explanation for this apparent fine tuning is the anthropic principle [1, 2, 3, 4, 5, 6, 7], that as observers we can observe only conditions (including the constants of physics) that permit our existence. However, it has been controversial what the deeper implications of this are.

One view is it is purely an accident or coincidence that the constants of physics have biophilic values, and that there is no deeper explanation. However, the fact that the cosmological constant is roughly 122 orders of magnitude smaller than the apparently simplest natural nonzero value for it (the Planck value) cries out for an explanation beyond pure coincidence, since the probability of such a remarkable coincidence from a random selection of the cosmological constant with a measure uniformly distributed over a range roughly the Planck value is extremely low, much less than the probability of having a monkey randomly type on a simple typewriter in one go,

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The cosmological constant is 10^(-122) in Planck units.
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A second view is that there are simple principles of physics, not specifically connected with life, that uniquely determine the constants to have the values they are observed to have. This view was commonly thought to be the case for the cosmological constant when it was believed to be zero (though no compelling simple principle was ever found that clearly implied that it would be zero). However, now that the cosmological constant has been found to have a very small positive value, it is hard to see how a simple principle of physics, independent of life, would uniquely fix that tiny nonzero value.

Perhaps a more common view among physicists today is the idea that there is a multiverse with a wide range of values for the constants of physics, and by the selection principle of observership (the weak anthropic principle), we find ourselves in the part of the multiverse where life is possible and/or relatively common (at

least compared to other parts of the multiverse) [7]. However, there is still considerable controversy over whether such a multiverse that would be necessary for this explanation really exists.

A fourth view is that there is some principle the fixes the constants of physics as they are in order that life would occur in the universe, perhaps in some maximal way. One version of this is the idea that "A theist would not be surprised if God had optimized the universe for life, not merely made life possible" [10]. The scientific form of this hypothesis would be that there is some biophilic property that is maximized by the constants of physics, so that the constants of physics are optimally fine tuned for life in some specific way.

The first view outlined above, pure coincidence, is hard to prove or disprove, though it would seem implausible and would be hard to maintain if any of the other three explanations were found to work well. The second idea, unique determination by simple principles of physics, could in principle attain strong evidence for it if such principles were found and were convincingly shown to lead to the observed constants of physics. It would be hard to disprove, since one could always believe that there were such principles that simply had not yet been discovered, but the more that scientists fail to find such principles, the less attractive this option will seem, particularly if some of the other options appear to work. The third view, of observer selection within a multiverse, is hard to prove or disprove directly, since it appears very difficult to obtain direct information about other possible parts of a multiverse. However, if a simple theory were developed that gives good statistical explanations for what we do observe and that also predicts a multiverse that we cannot directly observe, such a theory could become highly convincing (analogous to the prediction by general relativity of very high curvature in black-hole interior regions that cannot be directly observed). The fourth view, a biophilic principle of fine tuning for life, is also hard to prove or disprove, but if it is cast into the prediction that a certain calculable quantity is maximized, then it could also be scientifically testable.

In this paper I shall examine a particular variant of the fourth view, that the fraction of baryons that develops into living organisms is maximized by the observed constants of physics. This hypothesis is in principle falsifiable, and I shall argue that considerations of hypothetical variations of the cosmological constant give a very preliminary inconclusive hint that it may become falsified, since the fraction of baryons that condense into galaxies that in turn form living organisms would be higher if the cosmological constant were lower. This result thus gives a preliminary suggestion that there might eventually be evidence against optimal fine tuning for

life (or at least for maximizing the fraction of baryons that become living organisms) by such a biophilic principle.

However, email comments by Robert Mann [11], Michael Salem [12], and Martin Rees [13] have shown me that it is not at all clear that the very small increase in the fraction of baryons that would condense into galaxies if the cosmological constant were zero instead of its tiny observed positive value would also lead to an increase in the fraction of baryons that would go into life, since conceivably the small differences in the galaxies produced by lowering the cosmological constant to zero might also affect the fraction of baryons within galaxies that become life in a way that would overcompensate for the higher fraction of baryons condensing into galaxies.

One should also recognize, as Nicholas Beale [14] has emphasized to me, that the total number of baryons might depend on the cosmological constant in such as way as to make the total amount of life with a smaller cosmological constant lower, even if the increase in the fraction of baryons condensing into galaxies really does lead to an increase in the fraction of baryons incorporated into living organisms. Although I shall discuss the latter issue below, for the most part in this paper I am essentially ignoring it and merely considering the hypothesis that the cosmological constant is optimally fine tuned to maximize the fraction of baryons becoming living organisms.

A further caveat stressed by Robin Collins [15] and Beale [14] is that it is by no means clear that it would be the total amount of life that would be optimized, but perhaps intelligent life [17], "life that develops technology and science" [15], "life that is capable of real love" [14], "life that could discover the laws of physics" [14, 16], etc. So even if one could show that there is only a single universe with constants of physics that could be fine tuned for life but were not optimally fine tuned to maximize life, this by itself would not be evidence against the hypothesis that the constants are optimally fine tuned to maximize something having to do with life. However, to formulate a conjecture that in principle can be tested scientifically, it seems one needs to make it much sharper than can be justified by any such arguments, and in this paper it is only the very specific hypothesis that the constants of physics are optimally fine tuned to maximize the fraction of baryons forming living organisms that is considered. Here I am also making the very strong assumption that the cosmological constant can indeed be varied entirely independently of all the other constants of physics that may be responsible for all microphysics, including inflation, flatness, density fluctuations, baryosynthesis, nucleosynthesis, relative particle numbers, particle masses, coupling constants, stellar evolution, chemistry, geophysics, biology, Darwinian evolution by natural selection, life, complex life, intelligence, observers, observation, consciousness, etc.

2 The positive cosmological constant as a very preliminary suggestive hint of evidence against fine tuning for maximizing the fraction of baryons in living organisms or observers

Martel, Shapiro, and Weinberg [18], following upon previous ideas of Weinberg [19], have shown that the fraction of baryons that condense gravitationally into structures large enough to form living observers is a very sensitive function of the cosmological constant Λ that decreases rapidly if Λ is much larger than the observed value Λ_O (which is about 3.5×10^{-122} in Planck units, $\hbar = c = G = 1$). Therefore, if indeed there is a multiverse with a wide range of values of Λ that are fairly uniformly distributed near $\Lambda = 0$, the third view, observer selection within a multiverse, would be a good explanation for the observed value Λ_O.

However, here we are examining the alternative hypothesis from the fourth view, that a biophilic principle optimally fine tunes Λ to the value that maximizes the fraction of baryons that develop into life. Martel, Shapiro, and Weinberg [18] found that not only does the fraction of baryons condensing into galaxies drop steeply with Λ if it is much larger than Λ_O, but also that it is a decreasing function of Λ for all positive values. The simple reason, which does not depend on any of the details of the particular nonlinear model Martel, Shapiro, and Weinberg used, is that a positive cosmological constant gives a repulsion between distant particles that reduces the ordinary gravitational attraction and leads to less gravitational condensation of matter. Therefore, any positive cosmological constant decreases the fraction of baryons that condense to form galaxies and other structures that eventually may form living substructures.

As a consequence, if the fraction of baryons that form living organisms out of the baryons that condense into galaxies is independent of the fraction of baryons that condense into galaxies out of all baryons, no positive value of the cosmological constant (such as the observed value Λ_O) can maximize the fraction of baryons in life. Therefore, under the assumption of the previous sentence (that a fixed fraction of the baryons that condense into galaxies become living organisms), the observed positive value of the cosmological constant is evidence against the specific hypothesis of optimally fine tuning for life by a biophilic principle that would maximize the fraction of baryons that form living organisms or observers.

However, I have been informed by Robert Mann [11], Michael Salem [12], and Martin Rees [13] that it is plausible that the fraction of baryons within galaxies that form life depends on the cosmological constant (perhaps through the fraction

of baryons that condense into galaxies) in such a way that it might reverse what would otherwise be the increase in the fraction of all baryons forming life with a decrease of the cosmological constant below its observed value. For example, Mann [11] raised the possibility that if a larger fraction of baryons collapses, then perhaps statistically the gravitational bound states that form are more biophobic, with the stronger gravity of these structures making it less likely for life to survive. Salem [12] agreed that the collapse fraction of baryons is greater when the cosmological constant has a smaller magnitude but pointed out, "It seems plausible to me that the chance of a disruptive astrophysical event is larger if the rate of baryon accretion is larger." Similar but greater effects from a negative cosmological constant were noted in his earlier paper [20]. Rees [13] emphasized, "there are other astrophysical complexities: e.g. how much gas is retained in galaxies, despite the injection of energy from stellar winds and supernovae, which is then able to form later-generation stars containing heavy elements," and concluded that the small effect of the observed value of the cosmological constant on the fraction of baryons that condense into galaxies "is likely to be swamped by much bigger factors that we can't quantify," so that "it's not obvious that the trend is monotonic."

As Rees [13] later noted, "I agree with you that if lambda is below the critical value that suppresses galactic scale structure completely (and we know it is several times below that), nothing should depend very much on its value (unless the probability of life spreading through the galaxy happened to be exceedingly sensitive to some feature of galaxies that might be a function of lambda)." Therefore, the issue comes down to comparing several small effects. Originally I just considered the effect of the cosmological constant on the fraction of baryons that condense into galaxies, say F_G, and assumed that the fraction of baryons in galaxies that become life, say F_{LG}, is rather insensitive to the cosmological constant, so that the fraction of baryons that become life, say F_L, is proportional to F_G with a constant proportionality factor F_{LG} and hence decreases monotonically with the cosmological constant for positive values less than the observed value. That is, I was assuming that $F_L = F_G F_{LG}$ with F_{LG} rather independent of the fraction F_G that increases as Λ is reduced below its observed value Λ_0 (also assuming, as I shall continue to do, that life develops only from baryons that condense into galaxies).

Let $y = \Omega_\Lambda$, the ratio of the energy density of the cosmological constant to the critical density at the present observed value of the Hubble constant H_0 (which is assumed to be held fixed under a hypothetical variation in the cosmological constant, so that the cosmological constant is a fixed constant of proportionality multiplied by y, which I shall thus use as a measure of the cosmological constant). In what follows, the derivatives with respect to y are partial derivatives keeping

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all the other constants of physics fixed, but since in this paper I am only considering varying the cosmological constant with the other constants held fixed, they are also ordinary derivatives under this assumed restriction. Then $\partial(\ln F_L)/\partial y = \partial(\ln F_G)/\partial y + \partial(\ln F_{LG})/\partial y$. Simple reasoning that is confirmed by Martel, Shapiro, and Weinberg [18] (but which does not depend on their detailed assumptions) shows that $\partial(\ln F_G)/\partial y < 0$ for positive y at least, so if one neglects $\partial(\ln F_{LG})/\partial y$, then $\partial(\ln F_L)/\partial y < 0$ for positive y, implying that the maximum of $F_L(y)$ cannot be at the positive observed value of y. However, Mann [11], Salem [12], and Rees [13] noted effects that might make $\partial(\ln F_{LG})/\partial y > 0$. The open question is whether it could be that $\partial(\ln F_{LG})/\partial y > |\partial(\ln F_G)/\partial y| = -\partial(\ln F_G)/\partial y$, in which case $\partial(\ln F_L)/\partial y > 0$ for small y. Then since presumably $\partial(\ln F_L)/\partial y < 0$ for sufficiently large y (where $F_G(y)$ does become strongly suppressed by the repulsive effects of a large cosmological constant in a way that seems difficult to be compensated by any plausible increase in $F_{LG}(y)$), the maximum of $F_L(y)$ would be at positive y and in principle might possibly be at the observed value.

It is still not clear to me even what the sign of $\partial(\ln F_{LG})/\partial y$ is for very small positive values of the cosmological constant. One might naïvely guess that in a galaxy there would be a biophilic region where the metallicity is suitable for life, and where the density of stars is not so great that there are significant perturbations to the orbits of planets (so they can have rather stable temperatures). Presumably the centers of galaxies would not be very good for life from the latter effect, so perhaps it is not surprising that we do not find ourselves there. But then perhaps one could use as a surrogate of life, not merely the fraction F_G of baryons that condense into galaxies, but the fraction of baryons, say F_S, that condense into stars that are sufficiently similar to our sun in luminosity and metallicity and in not having too large a surrounding density of other stars. (Beale [14, 21] has alternatively suggested Habitable Earth-Like Planets or HELPs, though to maximize their expected number rather than to maximize the fraction of baryons that condense into them.) Even if galaxies are larger from the effects of more baryons condensing when y is smaller, it is not obvious to me that this fraction F_S of baryons that form suitable stars would be reduced, since even though the centers of such galaxies might be worse for life, conceivably the biophilic regions far away for the centers might have more suitable stars (and solar systems) for life if the galaxy as a whole has more baryons. But it is difficult to see how to estimate this effect, or other effects such as the amount of gas retained by the galaxies. Therefore, at present it seems one should just say that there are effects that might indeed make $\partial(\ln F_{LG})/\partial y > |\partial(\ln F_G)/\partial y|$ for small y, so that the maximum of $F_L(y)$, the fraction of baryons that form life, could be at positive y and conceivably even at the observed value.

3 Discussion and conclusions

If we could become convinced that for positive cosmological constant the derivative with respect to the cosmological constant (with all the other constants of physics held fixed) of the logarithm of the fraction F_{LG} of baryons in galaxies that become life is smaller than the absolute value of the (negative) derivative of the logarithm of the fraction F_G of all baryons that condense into galaxies, i.e., that $\partial(\ln F_{LG})/\partial y < |\partial(\ln F_G)/\partial y| = -\partial(\ln F_G)/\partial y$, then we would have evidence from the positive observed value of the cosmological constant that this constant of physics has not been optimally tuned to maximize the fraction F_L of baryons going into life. However, the fact that this inequality is uncertain implies that the present argument merely gives a preliminary inconclusive hint of evidence against optimal fine tuning of the cosmological constant for maximizing the fraction of baryons becoming life. Unless the cosmological constant is in fact optimally fine tuned in this way, it would seem that in principle one should be able to find that out by calculating $\partial(\ln F_L)/\partial y = \partial(\ln F_G)/\partial y + \partial(\ln F_{LG})/\partial y$ at the observed value of $y = \Omega_\Lambda$ and showing that it is nonzero. Therefore, the hypothesis of optimal fine tuning is in principle falsifiable, but unfortunately it appears to be premature at present to be able to perform the proper calculations or tests.

Another potential objection to the preliminary inconclusive hint of evidence against optimal fine tuning given here is that conceivably there are simple principles of physics that give relationships between the cosmological constant and other constants of physics that also affect life, so that if these principles are upheld (rather than allowing the cosmological constant to be varied independently of all the other constants of physics in the fine tuning as was assumed above), even if it can be shown that $\partial(\ln F_L)/\partial y \neq 0$ at the observed value of y with all of the other constants of physics held fixed, it might still possibly turn out that the cosmological constant and the other constants of physics in fact do maximize the fraction of baryons that become living organisms, subject to the constraints of the principles of physics that give the putative relationships between the constants. However, if these principles of physics allow variation of the constants, it is hard to see why they would not allow the cosmological constant to be varied rather independently from the other constants upon which life depends. For example, in the string landscape, it appears that the cosmological constant can vary rather independently of the other constants of physics.

A bigger objection, further emphasized by Beale [14], is it might seem unreasonable to maximize the fraction of baryons becoming life rather than just some measure of the totality of life itself. If the total number of baryons in the universe,

say B, were some function of Λ or y (with the other constants of physics held fixed at their observed values), then it would seem more reasonable to maximize something like the total number of baryons that form life (or perhaps life of some particular kind), say $L = BF_L$, rather than the fraction F_L. However, we do not know any such dependence of the total number of baryons on Λ, so in order to do their calculations, Martel, Shapiro, and Weinberg [18] left out that unknown dependence and just considered the fraction of baryons condensing into structures, and similarly I can also do little other than to leave it out. Nevertheless, that uncertainty is certainly cause for worry [22]. The situation seems to be even more ambiguous by the fact that the simplest estimates for the total number of baryons tend to be infinite. This leads us to the whole measure problem in cosmology [23, 24, 25, 26, 27, 20, 28, 29, 30, 31, 32, 33], which I admit has no universally accepted solution despite my own present favorite partial solution [33].

However, one can formulate the following crude rebuttal argument to the argument that it is plausible that B depends on Λ or y in just such a way that L is maximized at the observed value of y even if the maximum of $F_L(y)$ is not near the observed value of y: Since we have very little idea about $B(y)$, it seems that our uncertainty of $\ln |\partial(\ln B)/\partial y|$ is large. At the negative end, if the only dependence of B on y is from the very tiny relative change of the expansion rate of the universe during baryosynthesis, one would have $|\partial(\ln B)/\partial y| \ll |\partial(\ln F_L)/\partial y|$, so then we could ignore the variation of $B(y)$. At the positive end, one could have $|\partial(\ln B)/\partial y| >> |\partial(\ln F_L)/\partial y|$ for small $|y|$, in which case the maximum for $L(y)$ would presumably be for $|y|$ much larger than the observed value, again contrary to the hypothesis that the observed value maximizes $B(y)$. Assuming that $|\partial(\ln F_{LG})/\partial y|$ is not much larger than $|\partial(\ln F_G)/\partial y|$ that is small in comparison with unity, so that $|\partial(\ln F_L)/\partial y| \sim |\partial(\ln F_G)/\partial y| \ll 1$, for $B(y)$ to be maximized at the observed value would require that $\ln |\partial(\ln B)/\partial y| \sim \ln |\partial(\ln F_G)/\partial y|$, which seems a priori rather unlikely, given the large uncertainty in $\ln |\partial(\ln B)/\partial y|$.

It might be appropriate to note that although this paper has focused on the scientifically testable question of whether the constants of physics maximize a particular measure for life, it obviously also has theological implications. It could be taken as a preliminary inconclusive hint of negative evidence for theists who expect God to fine tune the constants of physics optimally for the fraction of baryons going into life [10] (though even these authors do not expect such an optimal tuning merely for the fraction of baryons going into life, or even for just the total amount of life without regard to its quality, so I have taken their writings as motivation for a much stronger hypothesis in order that it might be testable). However, for other theists, such as myself, even if it could be shown that the constants of physics are not optimally fine

tuned to maximize any reasonable measure of life in a single universe with just one set of constants, such evidence may simply support the hypothesis that God might prefer a multiverse with many sets of constants as the most elegant way to create life and the other purposes He has for His Creation [34].

In conclusion, the fact that the observed cosmological constant is positive may be taken to be a very preliminary inconclusive hint of evidence against a biophilic optimal fine tuning of it to maximize the fraction of baryons that develop into living organisms, since to maximize that fraction, the simplest (but highly uncertain) assumption would be that the fraction of baryons condensing into galaxies would need to be maximized, and for that the cosmological constant would instead need to be slightly negative.

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